High-order Quasi-Curl Conforming functions for Multiplicative Calderón Preconditioning of the EFIE

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Introduction

Method of moments (MoM)-based electric field integral equation (EFIE) solvers are widely used for analyzing time-harmonic electromagnetic scattering from perfect electrically conducting (PEC) surfaces. In the last decade, MoM-based EFIE solvers using high-order basis functions [1] have been shown to be more accurate as well as CPU and memory efficient than their zeroth-order predecessors leveraging Rao-Wilton-Glisson (RWG) functions [2]. Due to the spectral properties of the EFIE operator [3], and their effects on the MoM interaction matrices, the use of Calderón identities for preconditioning the EFIE has been studied extensively. In this context, Buffa-Christiansen (BC) basis functions [4] recently have been shown to permit a robust discretization of the Calderón preconditioned EFIE [5,6]. Unfortunately, the BC basis functions are zeroth-order in nature, and only can be used in conjunction with RWG basis functions. This limitation imposes a severe constraint on the accuracy and efficiency of present Calderón preconditioned EFIE solvers. In this paper, a new set of basis functions is presented, that extends the qualities of the (zeroth-order) BC functions to high-order.

Background

Consider a closed PEC surface S residing in a homogeneous medium with electric permittivity ϵ and magnetic permeability μ. The electric current $\boldsymbol{J}$ induced on S by the time-harmonic incident electric field $\boldsymbol{E}_{inc}$ satisfies the EFIE

$$\mathcal{T}(\boldsymbol{J}) = -\hat{\boldsymbol{n}}_r \times \boldsymbol{E}_{inc}, \tag{1}$$

Where the EFIE operator is defined as $\mathcal{T}(\boldsymbol{J}) = \mathcal{T}_s(\boldsymbol{J}) + \mathcal{T}_h(\boldsymbol{J})$, with

$$\mathcal{T}_s(\boldsymbol{J}) = \frac{ik}{4\pi}\hat{\boldsymbol{n}}_r \times \int_S \frac{e^{ik|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}\boldsymbol{J}(\boldsymbol{r}')ds', \tag{2}$$

and

$$\mathcal{T}_h(\boldsymbol{J}) = \frac{i}{4\pi k}\hat{\boldsymbol{n}}_r \times \int_S \nabla' \frac{e^{ik|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}\nabla'\cdot\boldsymbol{J}(\boldsymbol{r}')ds'. \tag{3}$$

Here $k = \omega\sqrt{\epsilon\mu}$ is the wave number, ω is the source's angular frequency, $i = \sqrt{-1}$, and $\hat{\boldsymbol{n}}_r$ is an outward directed unit vector normal to S at observer $\boldsymbol{r} \in S$. To solve EFIE (1) by the MoM, S is approximated by a mesh S_δ of planar or curvilinear triangles with minimum edge size δ, and $\boldsymbol{J}(\boldsymbol{r})$ is approximated as $\boldsymbol{J}(\boldsymbol{r}) \approx \sum_{n=1}^{N} I_n \boldsymbol{f}_n(\boldsymbol{r})$ where $\boldsymbol{f}_n(\boldsymbol{r})$, $n = 1,\ldots,N$ are order-p div-conforming Graglia-Wilton-Peterson (GWP) basis functions [1]. To obtain the expansion coefficients I_n, the above expression for $\boldsymbol{J}(\boldsymbol{r})$ is substituted into (1) and the resulting equation is tested with curl-confirming basis functions $\hat{\boldsymbol{n}}_r \times \boldsymbol{f}_n(\boldsymbol{r})$, yielding the $N \times N$ MoM EFIE system

$$\bar{\bar{\mathbf{Z}}}\,\bar{\mathbf{I}} = \bar{\mathbf{V}}, \tag{4}$$

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where $(\overline{\overline{\mathbf{Z}}})_{i,j} = \langle \hat{\boldsymbol{n}}_r \times \boldsymbol{f}_i, \mathcal{T}(\boldsymbol{f}_j) \rangle$, $(\overline{\mathbf{V}})_i = -\langle \hat{\boldsymbol{n}}_r \times \boldsymbol{f}_i, \hat{\boldsymbol{n}}_r \times \boldsymbol{E}_{inc} \rangle$, and $(\bar{\mathbf{I}})_j = I_j$. For large N, (4) only can be solved iteratively. Unfortunately, the spectral properties of the operator $\mathcal{T}$ [3] are such that the condition number of $\overline{\overline{\mathbf{Z}}}$ grows without bound as $\boldsymbol{J}(\boldsymbol{r})$ is better approximated ($\delta \to 0$ and/or $p \to \infty$). This problem can be solved by using the self-regularizing properties of $\mathcal{T}$ [5-7] expressed by the Calderón identities. The Calderón-preconditioned EFIE reads

$$\mathcal{T}^2(\boldsymbol{J}) = -\mathcal{T}(\hat{\boldsymbol{n}}_r \times \boldsymbol{E}_{inc}). \tag{5}$$

Since all singular values of the operator $\mathcal{T}^2$ accumulate at $-1/4$, the discretization of (5) leads to a well-conditioned MoM system, independently of δ and p. Standard discretization techniques which are suitable for stand-alone operators like $\mathcal{T}$ cannot be used straightforwardly when dealing with products like $\mathcal{T}^2 = \mathcal{T} \cdot \mathcal{T}$. On the one hand, the direct discretization of these products is not feasible since $\mathcal{T}(\boldsymbol{f}_n)$ is not available in closed form. On the other hand, when div/curl-conforming GWPs are used to discretize the source/test space of $\mathcal{T}$, the product of the discretized stand-alone operators would require the inversion of a mixed div/curl-conforming Gram matrix $(\overline{\overline{\mathbf{G}}})_{i,j} = \langle \hat{\boldsymbol{n}} \times \boldsymbol{f}_i, \boldsymbol{f}_j \rangle$, which is singular [4]. A fundamental feature of the product $\mathcal{T}^2 = \mathcal{T}_s^2 + \mathcal{T}_s\mathcal{T}_h + \mathcal{T}_h\mathcal{T}_s + \mathcal{T}_h^2$ is that $\mathcal{T}_h^2 = 0$. Any discretization technique for $\mathcal{T}^2$ must preserve this last identity, otherwise it will lead to a system that does not possess the spectral properties of the continuous operator $\mathcal{T}^2$. Several discretization techniques for $\mathcal{T}^2$ exist in the literature. Many of them call for the separate discretization of all terms in the operator product $\mathcal{T}^2$, this by means of an artificial annihilation of $\mathcal{T}_h^2$ and appropriate operational manipulations to compute the other three terms [7, 8]. Even though these techniques have been shown to give satisfactory results in frequency-domain simulations, they no longer preserve their qualities when dealing with time-domain marching on time EFIEs [6]. A different kind of discretization technique that uses BC basis functions [4] $\boldsymbol{f}^{BC}$ to systematically construct a multiplicative discretization of $\mathcal{T}^2$ was presented in [5]. This technique also was shown effective for dielectric structures [9] as well as for PEC structures in time-domain simulations [6]. When $\mathcal{T}^2$ is discretized using BC basis functions [5,6] the identity $\mathcal{T}_h^2 = 0$ is preserved upon discretization. Unfortunately, BC basis functions are zeroth-order in nature, and therefore can only be used with (zeroth-order) RWG basis functions. In what follows, a new set of basis functions is described, that extends the properties of the BC functions to high-order for use in conjunction with GWP basis functions in the construction of high-order Calderón preconditioned EFIE solvers.

Construction of the higher order quasi-curl conforming basis functions

The effectiveness of the technique proposed in [5] for discretizing $\mathcal{T}^2$ relies on two important properties of the BC basis functions: a) they are comprised of linear combinations of div-conforming RWG basis functions defined on the mesh $S_{\delta/2}$, a barycentric refinement of the original mesh S_δ [4,5,6], and resemble the curl-conforming basis functions defined on the original (coarse) mesh S_δ leading to a well-conditioned Gram matrix $(\overline{\overline{\mathbf{G}}}_{nfBC})_{i,j} = \langle \hat{\boldsymbol{n}}_r \times \boldsymbol{f}_i, \boldsymbol{f}_j^{BC} \rangle$. b) The dimension of the solenoidal (divergence-free) subspace of the space spanned by the BC basis functions matches the dimension of the rotational (curl$\neq 0$) subspace of the space spanned by the curl-conforming RWG basis functions. As explained in [6], this property leads to the preservation of the identity $\mathcal{T}_h^2 = 0$ upon discretization of $\mathcal{T}^2$.

The new high-order quasi-curl conforming basis functions are constructed such that: a) as a linear combination of order-*p* div-conforming GWP basis functions defined on $S_{\delta/2}$, they resemble the order-*p* curl-conforming GWP basis functions defined on S_δ. b) They span a space (denoted by $\widetilde{X}$) whose solenoidal subspace has the same dimension as the rotational subspace of the space spanned by the curl-conforming GWP basis functions defined on S_δ.

Let X be the space spanned by the N order-*p* div-conforming GWP basis functions defined on S_δ. X can be decomposed into a solenoidal subspace X_{sol} and its complement (non-solenoidal) subspace X_{nonsol} [10]. Elements in the basis of X_{sol} will be referred as "*Standard Loops*", while those in the basis of X_{nonsol}, as "*Standard Stars*". On its curl-conforming counterpart, a similar decomposition holds: If $\hat{\boldsymbol{n}}\times X$ denotes the space spanned by the curl-conforming GWP basis functions, then "$\hat{\boldsymbol{n}}\times$*Standard Loops*" span the irrotational subspace $\hat{\boldsymbol{n}}\times X_{sol}$, and "$\hat{\boldsymbol{n}}\times$*Standard Stars*" span the rotational subspace $\hat{\boldsymbol{n}}\times X_{nonsol}$. Similarly, if X^B denotes the space spanned by the N^B order-*p* div-conforming GWP basis functions defined on $S_{\delta/2}$, then "*Barycentric Loops*" span X^B_{sol}, "*Barycentric Stars*" span X^B_{nonsol}, and so on. With both spaces, X and X^B being decomposed in terms of "*loops*" and "*stars*", the elements in the solenoidal subspace of $\widetilde{X}$ are obtained as the columns of the matrix

$$\overline{\overline{\mathbf{P}}}_L = \left(\overline{\overline{\mathbf{L}}}_B^T \cdot \overline{\overline{\mathbf{G}}}_{ff}^B \cdot \overline{\overline{\mathbf{L}}}_B\right)^{-1} \cdot \left(\overline{\overline{\mathbf{L}}}_B^T \cdot \overline{\overline{\mathbf{G}}}_{nff}^B \cdot \overline{\overline{\mathbf{R}}} \cdot \overline{\overline{\mathbf{S}}}_S\right). \tag{6}$$

Similarly, the elements in the non-solenoidal subspace of $\widetilde{X}$ are obtained as the columns of the matrix

$$\overline{\overline{\mathbf{P}}}_S = \left(\overline{\overline{\mathbf{S}}}_B^T \cdot \overline{\overline{\mathbf{G}}}_{ff}^B \cdot \overline{\overline{\mathbf{S}}}_B\right)^{-1} \cdot \left(\overline{\overline{\mathbf{S}}}_B^T \cdot \overline{\overline{\mathbf{G}}}_{nff}^B \cdot \overline{\overline{\mathbf{R}}} \cdot \overline{\overline{\mathbf{L}}}_S\right). \tag{7}$$

Where: $\overline{\overline{\mathbf{L}}}_B$ is the matrix whose columns are the "*Barycentric loops*", $\overline{\overline{\mathbf{S}}}_B$ is the matrix whose columns are the "*Barycentric stars*", $\overline{\overline{\mathbf{L}}}_S$ is the matrix whose columns are the "*Standard loops*", $\overline{\overline{\mathbf{S}}}_S$ is the matrix whose columns are the "*Standard stars*", $(\overline{\overline{\mathbf{G}}}_{ff}^B)_{i,j} = \langle \boldsymbol{f}_i^B, \boldsymbol{f}_j^B \rangle$ is the barycentric Gram matrix, $(\overline{\overline{\mathbf{G}}}_{nff}^B)_{i,j} = \langle \hat{\boldsymbol{n}}_r \times \boldsymbol{f}_i^B, \boldsymbol{f}_j^B \rangle$ is the barycentric mixed Gram matrix, and $\overline{\overline{\mathbf{R}}}$ is the mapping from GWP basis functions defined on S_δ to GWP basis functions defined on $S_{\delta/2}$. All matrices listed above are sparse, and their inverses well-conditioned. Formulas (6)-(7) implement nothing but the L_2 projection of the basis of $\hat{\boldsymbol{n}}\times X_{nonsol}$ onto X^B_{sol} and $\hat{\boldsymbol{n}}\times X_{sol}$ onto X^B_{nonsol}, respectively.

Numerical Results

This section presents an example that demonstrates the effectiveness of the high-order quasi-curl conforming basis functions constructed here. Fig. 1 shows the error in bistatic RCS of a PEC sphere of radius $R = \lambda/10$ illuminated by an x-polarized plane wave propagating in the z direction. The RCS results are obtained for different orders and compared to the Mie series solution. Clearly, as the order increases, the error goes down. Table 1 shows the condition number of the EFIE and the Caderón-preconditioned EFIE for several orders and discretizations. For each order, the condition number of $(\mathcal{T}^2)_{dis}$ remains stable, independent of the discretization; whereas the condition number of $(\mathcal{T})_{dis}$ grows as the number of unknowns increases. The condition number of the mixed Gram matrix between quasi-curl conforming and curl-conforming GWPs (for simplicity, here denoted as G_{nfBC}) is also shown.

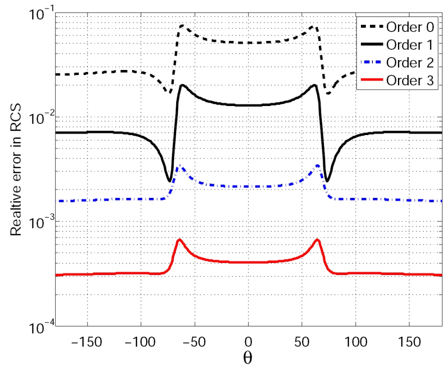

Fig. 1: Relative error in RCS in higher order Calderón-preconditioned EFIE.

Order 1	**# unknowns**	**160**	**510**	**1120**
	G_{nfBC}	19.69	32.26	42.91
	$(\mathcal{T})_{dis}$	408.47	2298.3	5476.3
	$(\mathcal{T}^2)_{dis}$	22.03	25.83	33.17
Order 2	**# unknowns**	**336**	**1071**	**2352**
	G_{nfBC}	55.84	80.9	88.73
	$(\mathcal{T})_{dis}$	3434.2	20914	46557
	$(\mathcal{T}^2)_{dis}$	62.69	93.5	83.63
Order 3	**# unknowns**	**576**	**1836**	
	G_{nfBC}	141.6	206.1	
	$(\mathcal{T})_{dis}$	29142	187130	
	$(\mathcal{T}^2)_{dis}$	228.6	294.3	

Table 1: Condition number for several orders and several mesh discretizations.

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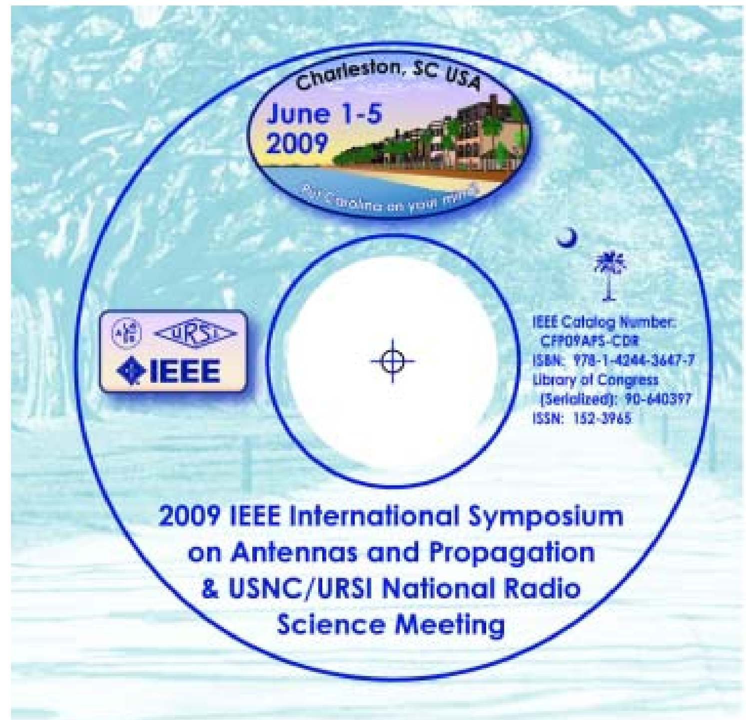

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